SOLUTION BY THE PROPOSER.

Let a = number of letters and spaces in heading.
b = number of spaces on 80 scale of right stop.

Then $\frac{80-a}{2}$ = number on 80 scale to begin heading. But by the method used this number is found as $\left\{\left(40-\frac{b}{2}\right)-\frac{a}{2}\right\}+\frac{b}{2}$.

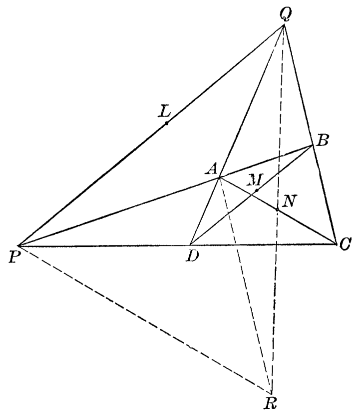
GEOMETRY.

418. Proposed by MRS. H. E. TREFETHEN, Waterville, Maine.

The difference of the squares of the two interior diagonals of a cyclic quadrilateral is to twice their rectangle as the distance between their mid-points is to the third diagonal.

SOLUTION BY C. N. SCHMALL, New York, N. Y.

Let $ABCD$ be a cyclic quadrilateral. Let the sides BA, CD meet in P, and the sides DA, CB in Q. Draw PQ. The resulting figure is a *complete quadri-*



lateral and PQ is the *third diagonal*. Let L, M, N, be the mid-points of the three diagonals. Then we have to prove that

$$\frac{\overline{AC}^2-\overline{BD}^2}{2AC\cdot BD}=\frac{MN}{PQ}.$$

It is well known that the points, L, M, N, are collinear. (See RUSSELL'S *Sequel to Elem. Geom.*, p. 154, ex. 5; or LACHLAN'S *Pure Geom.*, p. 93, § 151; or MILNE'S *Cross-Ratio Geom.*, p. 113, ex. 3.) Hence $MN = LN - LM$, or

$$\frac{MN}{PQ}=\frac{LN}{PQ}-\frac{LM}{PQ}. \tag{1}$$

Draw QN and prolong it to R, making $NR = QN$. Draw AR. Then $QCRA$

is a parallelogram. Hence QC is parallel to AR and

$$QC = AR. \tag{2}$$

Now, the triangles PAC, PDB, are similar; for $\angle PCA = \angle PBD$, both standing on the arc AD in a cyclic quadrilateral. Hence

$$AC : DB :: PA : PD. \tag{3}$$

Likewise, the triangles QCA, QDB are similar; and

$$AC : DB :: QC : QD. \tag{4}$$

From (3) and (4),

$$PA : PD :: QC : QD;$$

or

$$PD : QD :: PA : QC;$$

or, by (2),

$$PD : QD :: PA : AR. \tag{5}$$

Again,

$$\angle PDQ = \angle PAR, \tag{6}$$

for

$$\angle PDQ = 180° - \angle ADC = \angle ABC$$

and

$$\angle PAR = \angle ABC.$$

Hence, by (5) and (6) the triangles PDQ and PAR are similar; since they have an angle in each equal and the including sides proportional. Therefore,

$$PA : PD :: PR : PQ;$$

or

$$PA : PD :: 2LN : PQ. \tag{7}$$

By (3) and (7) we have

$$AC : BD :: 2LN : PQ. \tag{8}$$

In a similar way, by drawing QM and prolonging it to S (say), making $MS = QM$, and then drawing PS, we can show that

$$AC : BD :: PQ : 2LM. \tag{9}$$

From (8) and (9),

$$\frac{LN}{PQ} = \frac{1}{2}\frac{AC}{BD}, \qquad \frac{LM}{PQ} = \frac{1}{2}\frac{BD}{AC}.$$

Substituting these values in (1), we have

$$\frac{MN}{PQ} = \frac{1}{2}\left(\frac{AC}{BD} - \frac{BD}{AC}\right); \qquad \text{or} \qquad \frac{MN}{PQ} = \frac{\overline{AC}^2 - \overline{BD}^2}{2AC \cdot BD}.$$

CALCULUS.

333. Proposed by C. N. SCHMALL, New York City.

Evaluate $\int\int\int \sqrt{\dfrac{1 - (x^2 + y^2 + z^2)}{1 + x^2 + y^2 + z^2}}\, dx\, dy\, dz$, where $x^2 + y^2 + z^2 < 1$.